

Related-Key Attacks

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The Related-Key Model

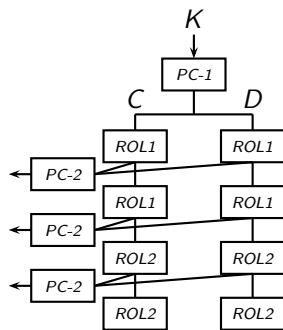
- ▶ Introduced by Biham and independently by Knudsen in 1993 [B93,K93]
- ▶ A block cipher is a keyed permutation, i.e.,
 $E : \{0, 1\}^n \times \{0, 1\}^k \rightarrow \{0, 1\}^n$ (or
 $E_k : \{0, 1\}^n \rightarrow \{0, 1\}^n$)
- ▶ Regular cryptanalytic attacks attack E by controlling the input/output of $E_k(\cdot)$
- ▶ In related-key attacks the adversary can ask to control k (chosen key attacks)
- ▶ This make look like a very strong notion, but the model allows for the adversary to control only the relation between keys

The Related-Key Model (cont.)

- ▶ In standard attacks, the adversary can query an oracle for E_k .
- ▶ In related-key attacks, the adversary can query the oracles E_{k_1}, E_{k_2}, \dots
- ▶ The adversary is either aware of the relation between the keys or **can choose** the relation
- ▶ This model which may look strong is actually not so far fetched:
 - ▶ Real life protocols allow for that (e.g., WEP)
 - ▶ When the block cipher is used as a compression function — the adversary may actually control the key
 - ▶ In some cases, there are properties so “strong”, that it is sufficient to have access to encryption under one key

DES's Key Schedule Algorithm

- ▶ The key is divided into two registers C and D (28-bit each)
- ▶ Each round both registers are rotated to the left (1 or 2 bits)
- ▶ 24 bits from C are chosen as the subkey entering $S1, S2, S3, S4$
- ▶ 24 bits from D are chosen as the subkey entering $S5, S6, S7, S8$



Round	1	2	3	4	5	6	7	8
Rotation	1	1	2	2	2	2	2	2
Round	9	10	11	12	13	14	15	16
Rotation	1	2	2	2	2	2	2	1

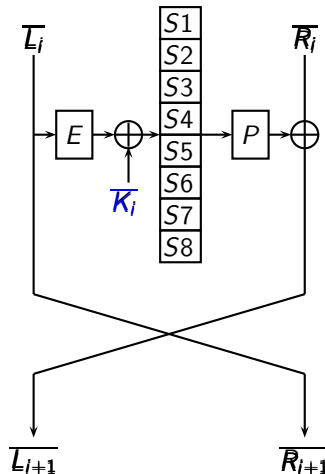
DES's Complementation Property

- ▶ If the key is bitwise complemented, so are all the subkeys

$$\begin{aligned} K &\rightarrow K_1, K_2, \dots, K_{16} \text{ and} \\ \overline{K} &\rightarrow \overline{K_1}, \overline{K_2}, \dots, \overline{K_{16}} \end{aligned}$$

- ▶ If the input to the round function is also bitwise complemented, the complementation is canceled
- ▶ In other words, the input to the S-boxes is the same **And the output of the S-boxes (and the round)**
- ▶ **DES's complementation property:**

$$DES_K(P) = \overline{DES_{\overline{K}}(\overline{P})}$$



Using the Complementation Property

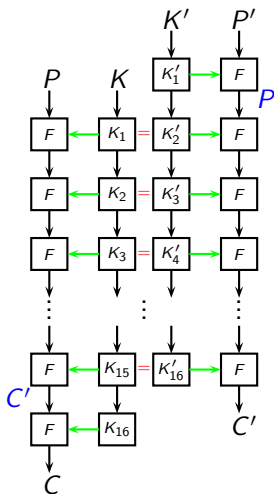
- ▶ Using the complementation property it is possible to speed up exhaustive key search of DES by a factor of 2
- ▶ The adversary asks for the encryption of P and \overline{P}
- ▶ Let $C_1 = E_K(P)$ and $C_2 = E_K(\overline{P})$, where K is the unknown key
- ▶ For each possible key k whose most significant bit is 0:
 - 1 Check whether $DES_k(P) = C_1$ (if yes, k is the key)
 - 2 Check whether $DES_k(\overline{P}) = C_2$ (if yes, \overline{k} is the key)

Note that $\overline{DES_k(P)} = C_2 \Rightarrow \overline{C_2} = DES_k(P)$.

As $C_2 = DES_K(\overline{P})$, then $\overline{DES_K(\overline{P})} = DES_k(P)$, i.e., $K = \overline{k}$

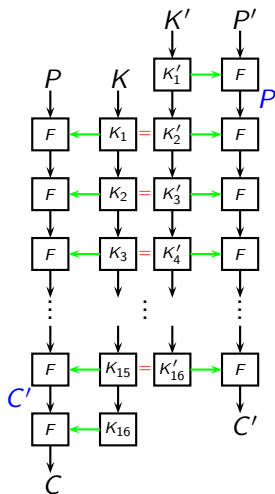
A Related-Key Attack on a Slightly Modified DES

- Assume that all the rotations in the key schedule are all by 2 bits to the left
- Consider two keys K and K' , such that the subkeys produced by the key schedule algorithm satisfy $K_i = K'_{i+1}$ (i.e., $K_1 = K'_2, K_2 = K'_3, \dots$)
- Then the first 15 rounds of encryption under K are just like the last 15 rounds of encryption under K'



A Related-Key Attack on a Slightly Modified DES

- ▶ Let $P = F_{K'_1}(P')$
- ▶ Due to the equality between the functions, P and P' share 15 rounds of the encryption
- ▶ Thus, $C = F_{K_{16}}(C')$
- ▶ Given (P, C) and (P', C') , deducing K'_1 and K_{16} (given DES's round function) is easy



A Related-Key Attack on a Slightly Modified DES

- ▶ Ask for the encryption of 2^{16} plaintexts $P'_i = (A, x'_i)$ under K' . Let $C'_i = E_{K'}(P'_i)$
 - ▶ Ask for the encryption of 2^{16} plaintexts $P_j = (y'_j, A)$ under K . Let $C_j = E_K(P_j)$
- 1 By birthday arguments there is a pair of values P'_i which is encrypted under one round to P_j . From this point forward, they are “evolving” together, and thus,
$$C_j = F_{K_{16}}(C'_i)$$
 - 2 From Feistel properties, that means that the left half of C'_i is equal to the right half of C_j

A Related-Key Attack on a Slightly Modified DES

- ▶ Search for a pair of ciphertexts C'_i and C_j such that the left half of C'_i is equal to the right half of C_j
- ▶ Deduce that $P_j = F_{K'_1}(P'_i)$ and that $C_j = F_{K_{16}}(C'_i)$, and retrieve the key
- ▶ This pair is called *a related-key plaintext pair*
- ▶ Using this pair it is easy to deduce K'_1 and K_{16} (which are also share bits between themselves)

Data complexity: 2^{16} CPs under two related-keys (the relation was chosen by the adversary)

Time complexity: 2^{17} encryptions (the analysis phase is very efficient)

A Second Attack on a Slightly Modified DES

- ▶ It is possible to use 2^{34} encrypted under **one** key, and still use the above property
- ▶ The idea is to generate a related-key plaintext pair by using one key, and then, trying to identify the pair
- ▶ Pick P_0 at random, and let $P_1 = \overline{P_0}$
- ▶ Define the set of plaintexts $\{P_{i,t} = (P_{iR}, P_{iL} \oplus t)\}$ for all possible t 's, P_0 , and P_1
- ▶ Define the set of plaintexts $\{P_{i,t}^* = (P_{iR} \oplus k, P_{iL})\}$ for all possible t 's, P_0 , and P_1
- ▶ Ask for the encryption of $P_i, P_{i,t}$ and $P_{i,t}^*$

Preprocessing

- ▶ Consider 28-bit half subkeys such the MSB is 0
- ▶ Define the *next* operation, which takes a subkey, rotates it to the left by 2 bits. In case the obtained subkey has its MSB equal to 1, the subkey is complemented
- ▶ After at most 14 *next* operations, a cycle is found
- ▶ Thus, in the preprocessing we generate a list of one half subkey L_i from every such cycle (the list contains about $2^{23.2}$ values)

The Attack

- ▶ Given C_i , $C_{i,t}$ and $C_{i,t}^*$:

- 1 For each key of the form (L_j, L_k) , generate all keys

$$K' = (ROL_m(L_j) || ROL_n(L_k)) \text{ or } K' = (ROL_m(L_j) || ROL_n(\overline{L_k}))$$

where $m \in \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26\}$
and $n \in \{0, 6, 12, 18, 24\}$

- 2 Encrypt P_0 under each K' , and obtain $C' = E_{K'}(P_0)$
- 3 If $C' = C_0$ or $C' = \overline{C_1}$, deduce that the original key to is K' or $\overline{K'}$, respectively

The Attack

- ▶ **4** Assume that the output of the F function in the first round is t (when using the key K' to encrypt P_0). If C'_L is equal to some $C_{0,tR}$ or $\overline{C_{1,tR}}$, “continue” the encryption of P_0 to the 17th round. If the result C'' is $C_{0,t}$ or $\overline{C_{1,t}}$, then the key is

$$ROL_2(K'_R) || ROL_2(K'_L),$$

or its complement, respectively

- 5** “Decrypt” P_0 one round, and let the output of the F function in the 0th round be t (when using the key K' to encrypt P_0). If the intermediate encryption value of P_0 in the 15th round is $C_{0,k}^*$ or $\overline{C_{1,k}^*}$, then the key is

$$ROR_2(K'_R) || ROR_2(K'_L),$$

or its complement, respectively

Final Remarks

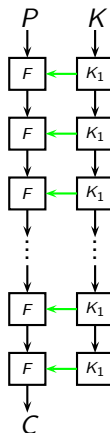
- ▶ The set of plaintexts $P_{i,t}$ is actually the set of possible related-key plaintext pairs where the shared rounds are 2–16
- ▶ The set of plaintexts $P_{i,t}^*$ is actually the set of possible related-key plaintext pairs where the shared rounds are 1–15
- ▶ **Homework:** Check that we covered all possible keys

Data Complexity: 2^{34} chosen plaintexts

Time Complexity: P_0 is encrypted under about 1/6 of the keys — $1.4 \cdot 2^{53}$ encryptions

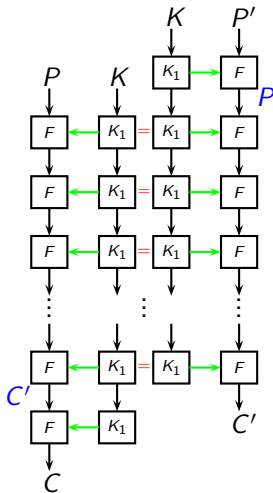
The Slide Attack

- ▶ Presented by Biryukov and Wagner in 1999
- ▶ Can be applied to ciphers with the same keyed permutation
- ▶ Independent of the number of rounds of the cipher
- ▶ To some extent, this attack is a related-key plaintext attack when the key is its own related-key



An Example — Slide Attack on 2K-DES

- ▶ Consider a variant of DES with $2r$ rounds, where the subkeys are $(K_1, K_2, K_1, K_2, \dots, K_1, K_2)$
- ▶ This variant has 96-bit key, and if r is large enough, no conventional attacks apply



A Related-Key Attack on a 2K-DES (cont.)

- ▶ Take 2^{32} known plaintexts, P_i (and their corresponding ciphertexts C_i)
- ▶ Let $f_{K_1, K_2}(\cdot)$ be two rounds of DES with the subkeys K_1 and K_2
- ▶ Then, the data set is expected to contain two plaintexts P_i and P_j such that $f_{K_1, K_2}(P_i) = P_j$ and $f_{K_1, K_2}(C_i) = C_j$ (denoted as a *slid pair*)

How do you Find the Slid Pair?

- ▶ Generally speaking, the best way to find the slid pairs is to try all of them
- ▶ So in this attack, the adversary considers each pair (P_i, P_j) (there are 2^{64} pairs, as the pair is ordered)
- ▶ For each pair, the adversary has two equations to solve:

$$f_{K_1, K_2}(P_i) = P_j; \quad f_{K_1, K_2}(C_i) = C_j$$

- ▶ This can be done very easily
- ▶ For each solution (if exists), verify the suggested key
- ▶ Time complexity — 2^{64} times solving the above set
- ▶ A possible improvement: Guess some part of K_1 (or K_2) which gives filtering on the pairs, and then there are less pairs to analyze

How do you Find the Slid Pair? (cont.)

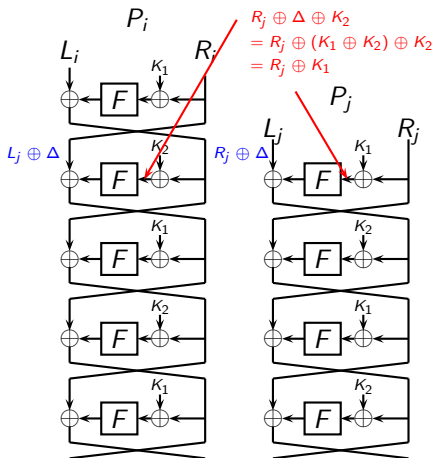
- ▶ This leads to a very interesting approach in block ciphers cryptanalysis
- ▶ To break a cipher X (to find the secret key), we need a slid pair
- ▶ To find this slid pair, we take many candidate pairs
- ▶ For each candidate pair, we analyze which key it suggests
- ▶ Then, if the key suggested is correct we found the slid pair. . . . which is what we need for finding the right key

Summary of the Slide Attack

- ▶ Independent of the number of rounds
- ▶ Generation of a slid pair in $O(2^{n/2})$ known plaintexts (or $2^{n/4}$ for Feistel block ciphers)
- ▶ Works if $F_K(P_i) = P_j, F_K(C_i) = C_j$ is sufficient for finding K

Complementation Slide Attack

- ▶ Consider 2K-DES
- ▶ Let $\Delta = K_1 \oplus K_2$
- ▶ Consider two plaintexts P_i, P_j such that if $X = f_{K_1}(P_i)$ then $X_i = P_j \oplus (\Delta, \Delta)$
- ▶ This relation remains until $C_j = f_{K_2}(C_i) \oplus (\Delta, \Delta)$

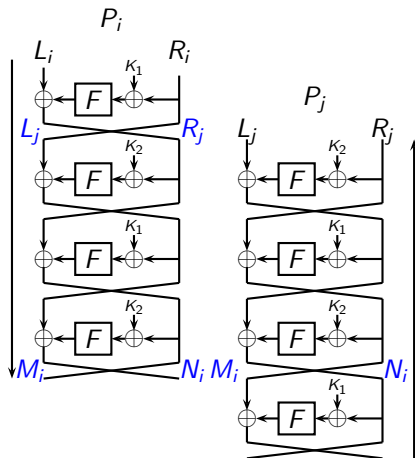


Complementation Slide Attack

- ▶ As half of the data is unchanged by $f(\cdot)$, the identification of slid pairs is easier
- ▶ Starting with 2^{32} known plaintexts, and use the filter condition on the differences (right half of P_i XOR the left half of P_j is equal to the right half of C_i XOR the left half of C_j) to discard most of the wrong candidate keys
- ▶ There is a small technicality here that makes the attack fail. If you recall, the difference in the data words is of 32 bits, and of the subkey is in 48-bit words
- ▶ Hence, this attack works, only if Δ is a legitimate output of $E(\cdot)$ of DES (i.e., the actual difference in the plaintext is $E^{-1}(\Delta)$)

Slide Attack with a Twist

- ▶ Consider encryption and decryption in a Feistel block cipher
- ▶ They are the same up to the order of subkeys
- ▶ Now, consider 2K-DES, with one round slide in the encryption direction and the decryption direction. . .
- ▶ Given 2^{32} known plaintexts, it is possible to find a twisted slid pair and repeat the analysis.



Slide Attack with a Twist (cont.)

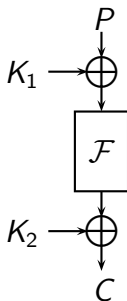
- ▶ This time, it is possible to analyze only one subkey (K_1), as the relations are

$$f_{K_1}(N_i) = C_j \oplus M_i; \quad f_{K_1}(R_i) = R_j \oplus L_i.$$

- ▶ This allows applying a chosen plaintext and ciphertext attacks with 2^{16} of each
- ▶ The adversary asks for the encryption of (A, x) and the decryption of (A, y)
- ▶ Note that this variant *actually works*
- ▶ And do note that you can combine the two techniques

The Even-Mansour Block Cipher

- ▶ Suggested by Even and Mansour in 1991, as a generalization of the DESX approach
- ▶ Apparently, even if you know the internal key of DESX, the system is still secure
- ▶ Main idea: Change the keyed permutation in the middle to an n -bit pseudo-random permutation \mathcal{F}
- ▶ Block size: n bits, Key size: $2n$ bits



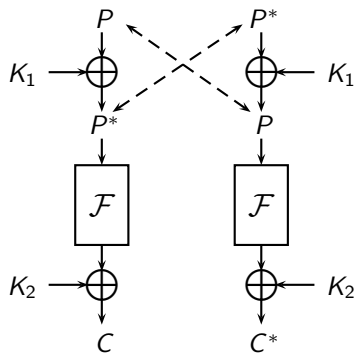
$$EM_{K_1, K_2}^{\mathcal{F}}(P) = \mathcal{F}(P \oplus K_1) \oplus K_2$$

Security of the Even-Mansour Scheme

- ▶ A simple attack that requires 2 plaintext/ciphertext pairs and 2^n time (so security is n -bits at most)
- ▶ There is a **proof** that any attack that uses D plaintext/ciphertext pairs, and T queries to \mathcal{F} , has success rate of $O(DT/2^n)$
- ▶ There is a differential attack that offers this tradeoff [D92]
- ▶ There is also a slide with a twist attack that uses $2^{n/2}$ data and time

Slide with a Twist Attack on Even-Mansour

- ▶ Consider two plaintexts P and P^* such that $P^* = P \oplus K_1$
- ▶ The inputs to \mathcal{F} are swapped, which means that so does the outputs
- ▶ Hence, $C \oplus C^* = \mathcal{F}(P) \oplus \mathcal{F}(P^*)$
- ▶ So the attack starts with $2^{n/2}$ plaintexts P_i , each is encrypted to the corresponding C_i , and a collision in the values of $C_i \oplus \mathcal{F}(P_i)$ is expected to suggest a slid pair



Slide with a Twist Attack on Even-Mansour

- ▶ The attack requires $D = 2^{n/2}$ known plaintexts
- ▶ To generate the table, $T = 2^{n/2}$ additional queries to \mathcal{F} are made
- ▶ The success rate is the probability of having a slid pair, which is quite high
- ▶ We note that having even slightly less than $O(2^{n/2})$ plaintexts results in the failure of the attack
- ▶ So this attack satisfies the bound, but at the same time, offers no tradeoff

Motivation

- ▶ The slide attack requires one slid pair to work
- ▶ To find such a pair, we need at least $2^{n/2}$ known plaintexts
- ▶ If we are given less data, can we somehow compensate for the lack of slid pairs with some computation?

SlideX Attack on Even-Mansour

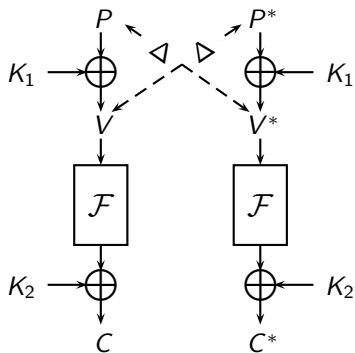
- ▶ Consider two plaintexts P and P^* such that $P^* = P \oplus K_1 \oplus \Delta$
- ▶ Then:

$$\begin{aligned} EM_{K_1, K_2}^{\mathcal{F}}(P) &= \mathcal{F}(P \oplus K_1) \oplus K_2 \\ &= \mathcal{F}(P^* \oplus \Delta) \oplus K_2 \end{aligned}$$

$$\begin{aligned} EM_{K_1, K_2}^{\mathcal{F}}(P^*) &= \mathcal{F}(P^* \oplus K_1) \oplus K_2 \\ &= \mathcal{F}(P \oplus \Delta) \oplus K_2 \end{aligned}$$

- ▶ Hence,

$$EM_{K_1, K_2}^{\mathcal{F}}(P) \oplus \mathcal{F}(P \oplus \Delta) = EM_{K_1, K_2}^{\mathcal{F}}(P^*) \oplus \mathcal{F}(P^* \oplus \Delta)$$



SlideX Attack on Even-Mansour (cont.)

- ▶ We define a SlideX pair, as a pair which actually satisfies the required relation $P = P^* \oplus K_1 \oplus \Delta$
- ▶ To check for the SlideX pair, we take the D plaintext/ciphertext pairs (P_i, C_i) , and for each Δ guess, we construct a table of all values $C_i \oplus \mathcal{F}(P_i \oplus \Delta)$
- ▶ The trick here, is that we check $O(D^2)$ pairs by each such guess of Δ
- ▶ Hence, we repeat the construction of the table $O(2^n/D^2)$ times, each time with D calls to \mathcal{F} , or $T = O(2^n/D)$ times in total

And we're done!

SlideX vs. Slide (with a Twist)

- ▶ The attack can work with any given amount of data
- ▶ As a SlideX pair is actually a SlideX tuple (with respect to some Δ), we can increase the number of Δ 's to compensate for the reduced data
- ▶ Additionally, we just need to store $O(D)$ values, so if $D \ll 2^{n/2}$, we can use a significantly smaller amount of memory

Related Work

- ▶ Finding more slid pairs [F01]: continuing to encrypt a slid pair offers another slid pair
- ▶ Reflection attacks [KM07,K08]: targets Feistel ciphers with key schedule such that $K_{r+t} = K_{r-t-1}$ (symmetry around K_r)
- ▶ Identification of slid pairs based on cycles [BDK07]

Related-Key Differential Attacks

- ▶ Consider the complementation property of DES:

$$DES_K(P) = \overline{DES_{\overline{K}}(\overline{P})}$$

- ▶ This equality can be rewritten as:

$$DES_K(P) \oplus DES_{\overline{K}}(\overline{P}) = FFFF \ FFFF \ FFFF \ FFFF_x$$

- ▶ Does this look familiar?
- ▶ This motivated Kelsey, Schneier and Wagner to introduce related-key differentials

Related-Key Differentials (cont.)

- ▶ The probability of regular differential is:

$$\Pr_{P,K}[E_K(P) \oplus E_K(P \oplus \Delta P) = \Delta C]$$

- ▶ The probability of related-key differential is:

$$\Pr_{P,K}[E_K(P) \oplus E_{K \oplus \Delta K}(P \oplus \Delta P) = \Delta C]$$

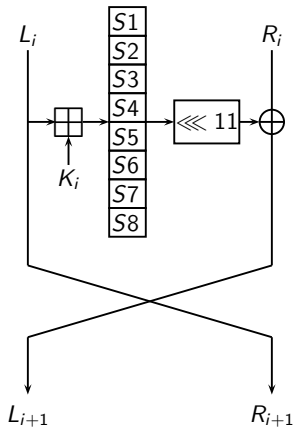
- ▶ The key difference leads to subkey differences, that may be used to cancel the differences in the input to the round function
- ▶ The reminder of the differential attack using a related-key attack is quite the same (up to the use of two keys)
- ▶ Usually, the key relation is by a difference, but other relations may be used as well*

*Note that the relation $K' = K \wedge Const$ and $K' = K \vee Const$, for any constant $Const$, allow for a trivial key recovery attack.

The Block Cipher GOST

- ▶ The Soviet/Russian block cipher standard (GOST 28147-89)
- ▶ 64-bit block, 256-bit key, 32 rounds
- ▶ S-boxes: 4×4 . Implementation specific
- ▶ Key schedule very simple, take $K = (K_1, K_2, \dots, K_8)$:

Round	1	2	3	4	5	6	7	8
Subkey	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8
Round	9	10	11	12	13	14	15	16
Subkey	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8
Round	17	18	19	20	21	22	23	24
Subkey	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8
Round	25	26	27	28	29	30	31	32
Subkey	K_8	K_7	K_6	K_5	K_4	K_3	K_2	K_1



Related-Key Differentials in GOST

- ▶ Flipping the MSBs of all key words, flips the MSB of all the subkeys
- ▶ Flipping the two MSBs of the plaintext words, leads to the same input entering the S-boxes in all rounds
- ▶ Thus, under a key difference $(80000000_x, 80000000_x, \dots, 80000000_x)$ the plaintext difference $(80000000_x, 80000000_x)$ leads to ciphertext difference $(80000000_x, 80000000_x)$ with probability 1
- ▶ Can speed up exhaustive search by a factor of 2 (like in DES)
- ▶ Or for a very simple distinguishing attack (with 2 chosen plaintexts)

Recovering the Key in GOST in a Related-Key Attack

- ▶ For a differential key recovery attack we need a differential with nontrivial probability
- ▶ Pick $\Delta K = (40000000_x, 40000000_x, \dots, 40000000_x)$
- ▶ An input difference $\Delta = (40000000_x, 40000000_x)$ remains unchanged after one round with probability $1/2$
- ▶ Thus, it is easy to build a 30-round related-key differential with probability 2^{-30} for GOST
- ▶ Then, GOST can be attacked using standard differential techniques

The Differences from Regular Differentials

- ▶ Despite the above there are few subtle differences between regular differentials and related-key differentials
- ▶ The amount of possible pairs, for example. In a one-key scenario, for a given input difference there are 2^{n-1} possible distinct pairs (n being the block size). In two-key scenario — 2^n
- ▶ Consider an input difference to an s -bit round function. Once the key is fixed, for any given input difference, there are at most 2^{s-1} output differences. In the related-key model there are 2^s (if there is a key difference, of course)

Concluding Remarks — Related Key Attacks

- ▶ Almost any statistical attack has a related-key variant:
 - ▶ Related-key impossible differential attack
 - ▶ Related-key boomerang/amplified boomerang/rectangle attack
 - ▶ Related-key linear attack
 - ▶ Related-key ...
- ▶ There are several works that actually run a statistical related-key attack inside a related-key attack