

Session 2: The Yao and BMR Protocols for Secure Computation

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The Yao and BMR Protocols

- Yao presented the first protocol for secure (**two-party**) computation
- Yao's protocol was followed by several protocols for the **multi-party** setting
 - Goldreich-Micali-Wigderson (GMW)
 - Ben Or-Goldwasser-Wigderson (BGW), Chaum-Crepeau-Damgård (CCD)
- Beaver-Micali-Rogaway (BMR) presented a multi-party protocol using a similar approach to Yao's, and with only $O(1)$ communication rounds.

Yao's Protocol

Yao's Protocol

- **A protocol for general secure two-party computation**
 - Constant number of rounds
 - The basic protocol is secure only for semi-honest adversaries
 - Many applications of the methodology beyond secure computation
- **General secure computation**
 - Can securely compute *any* functionality
 - Based on a representation of the functionality as a **Boolean circuit**

Representing functions as Boolean circuits???

- In some cases the circuits are small
 - Adding numbers
 - Comparing numbers
 - Multiplying numbers?
 - Computing AES?
 - Working with indirect addressing ($A[i]$) ?
- We can efficiently do secure computation of millions and billions of gates

Basic ideas

- ***A plain circuit is evaluated by***
 - Setting values to its input gates
 - For each gate, computing the value of the outgoing wire as a function of the wires going into the gate
- **Secure computation:**
 - No party should learn the values of any internal wires
- **Yao's protocol**
 - A compiler which takes a circuit and transforms it to a circuit which hides all information but the final output

Outline

- **Garbled circuit**

- An encrypted circuit together with a pair of keys (k_0, k_1) for every wire so that for any gate, given one key on every input wire:

- It is possible to compute the key of the corresponding gate output
- It is impossible to learn anything else

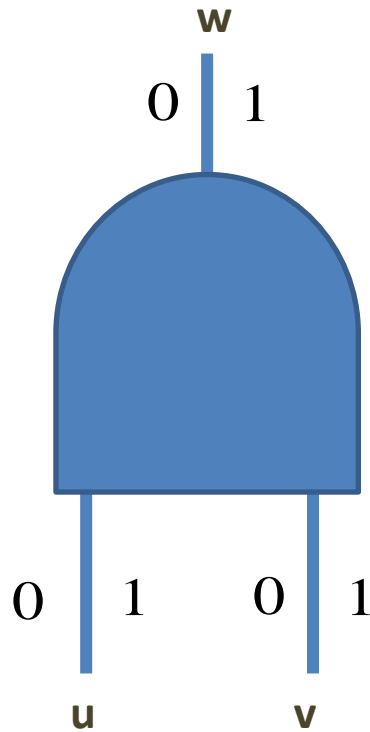
- **Tool: oblivious transfer**

- Input: sender has x_0, x_1 ; receiver has b
- Receiver obtains x_b only
- Sender learns nothing

A Garbled Circuit

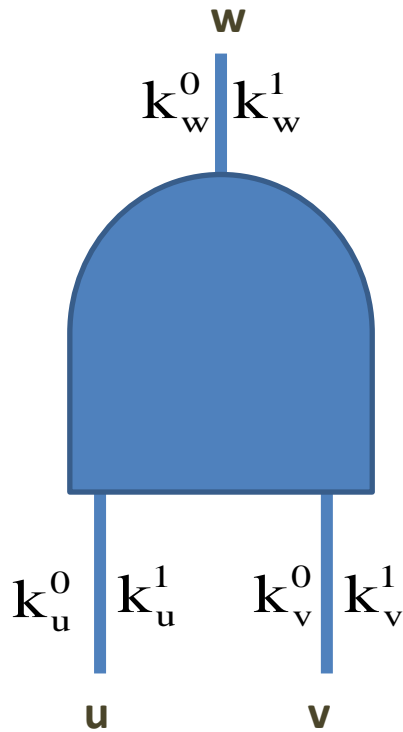
- For the entire circuit, assign independent random values/keys to each wire (key k_0 for 0, key k_1 for 1)
 - These keys are also called “garbled values”
- Encrypt each gate, so that given one key for each input wire, can compute the appropriate key on the output wire

An AND Gate



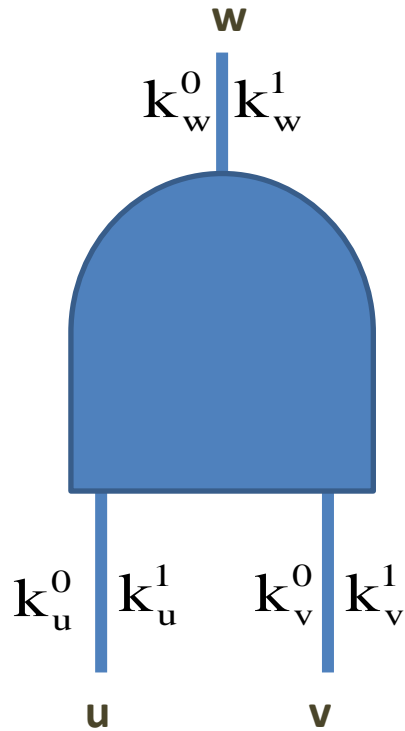
u	v	w
0	0	0
0	1	0
1	0	0
1	1	1

An AND Gate with Garbled Values



u	v	w
k_u^0	k_v^0	k_w^0
k_u^0	k_v^1	k_w^0
k_u^1	k_v^0	k_w^0
k_u^1	k_v^1	k_w^1

A Garbled AND Gate



u	v	w
k_u^0	k_v^0	$E_{k_u^0}(E_{k_v^0}(k_w^0))$
k_u^0	k_v^1	$E_{k_u^0}(E_{k_v^1}(k_w^0))$
k_u^1	k_v^0	$E_{k_u^1}(E_{k_v^0}(k_w^0))$
k_u^1	k_v^1	$E_{k_u^1}(E_{k_v^1}(k_w^1))$

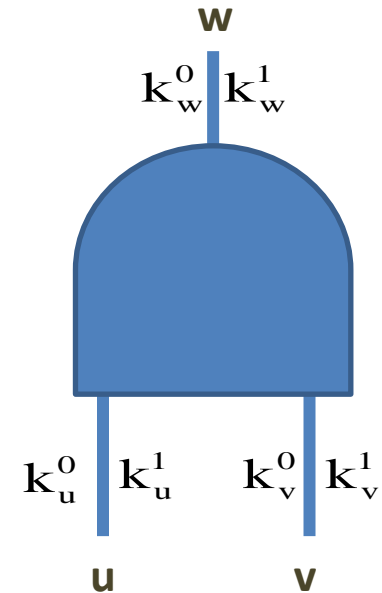
A Garbled AND Gate

- The actual garbled gate

in permuted order

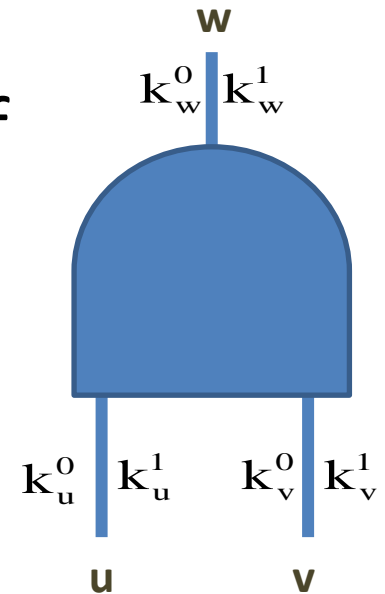
$$\left\{ \begin{array}{l} E_{k_u^1}(E_{k_v^0}(k_w^0)) \\ E_{k_u^0}(E_{k_v^1}(k_w^0)) \\ E_{k_u^1}(E_{k_v^1}(k_w^1)) \\ E_{k_u^0}(E_{k_v^0}(k_w^1)) \end{array} \right.$$

- Given K_u^0 and K_v^1 can obtain only K_w^0
- Furthermore, since the order of the rows is permuted, the party has no idea if it obtained the 0 or 1 key



Output Translation

- If the gate is an output gate, also need to provide the “decryption” of the output wire
- Output translation table:
 $[(0, k_w^0), (1, k_w^1)]$
- (Note: this table is insecure if the wire w is used as an input wire to any other gate. It is better to use a table of the form $[(0, H(k_w^0)), (1, H(k_w^1))]$ but this complicates the security proof.)



Constructing a Garbled Circuit

- **Given a Boolean circuit**
 - Assign garbled values to all wires
 - Construct garbled gates using the garbled values
- **Central property:**
 - Given a garbled value for each input wire, can compute the entire circuit, and obtain garbled values for the output wires
 - Given a translation table for the output wires, can obtain output
 - Nothing but the final output is learned!

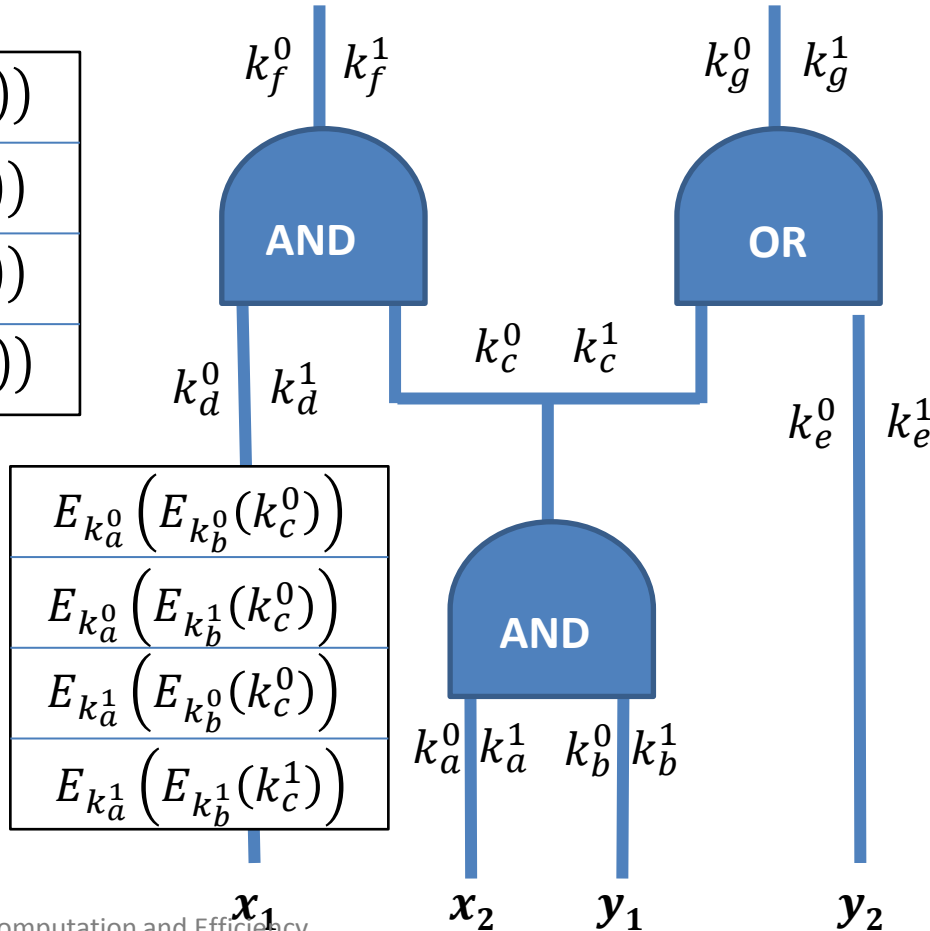
An Example Circuit

$[(0, k_f^0), (1, k_f^1)]$

$[(0, k_g^0), (1, k_g^1)]$

$E_{k_d^0}(E_{k_c^0}(k_f^0))$
$E_{k_d^0}(E_{k_c^1}(k_f^0))$
$E_{k_d^1}(E_{k_c^0}(k_f^0))$
$E_{k_d^1}(E_{k_c^1}(k_f^1))$

$E_{k_e^0}(E_{k_g^0}(k_g^0))$
$E_{k_e^0}(E_{k_g^1}(k_g^1))$
$E_{k_e^1}(E_{k_g^0}(k_g^1))$
$E_{k_e^1}(E_{k_g^1}(k_g^1))$



$E_{k_a^0}(E_{k_b^0}(k_c^0))$
$E_{k_a^0}(E_{k_b^1}(k_c^0))$
$E_{k_a^1}(E_{k_b^0}(k_c^0))$
$E_{k_a^1}(E_{k_b^1}(k_c^1))$

Computing a Garbled Circuit

- **How does the party computing the circuit know that it decrypted the “correct” entry?**
 - A gate table has four entries in permuted order
 - The keys known to the evaluator can decrypt only a single entry, but symmetric encryption may decrypt “correctly” even with incorrect keys
- **Two possibilities** (actually many...)
 - Add redundant zeroes to the plaintext; only correct keys give redundant block
 - Add a bit to signal which ciphertext to decrypt

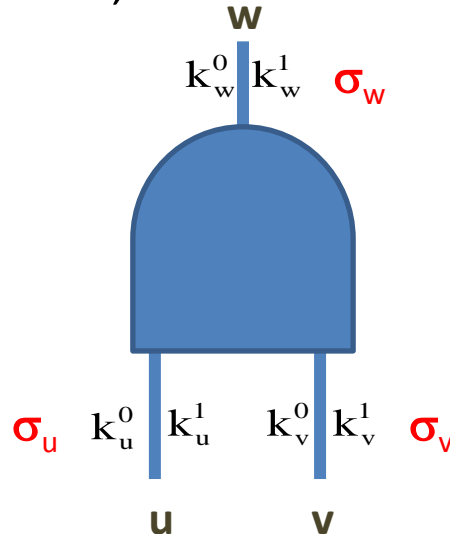
Computing a Garbled Circuit

- **Option 1:**

- Encryption: $E_K(m) = [r, F_K(r) \oplus (m \parallel 0^n)]$
 - By the pseudo-randomness of F , the probability of obtaining 0^n with an incorrect K is negligible
- any drawbacks?*

- **Option 2:**

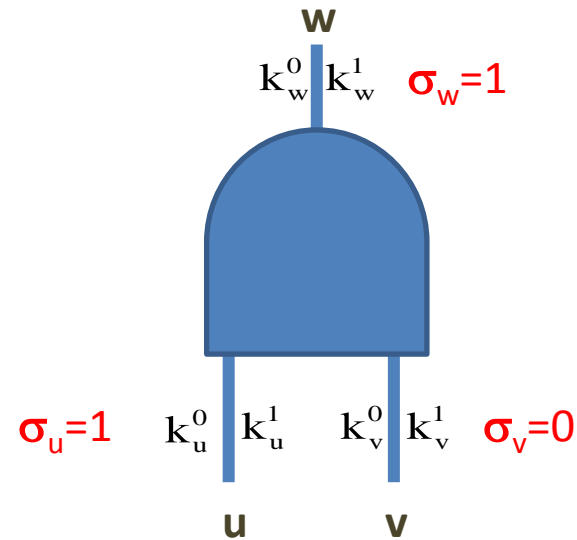
- For every wire, choose a **random signal bit** together with the keys



- Each wire has an “internal value” bit which must be kept secret
- It also has an “external value” bit which the evaluator can see
- External value is equal to **(internal value xor signal bit)**

Computing a Garbled Circuit with a Signal Bit

- The actual garbled gate table ordered based on external values
- | |
|---------------------------------------------------------------|
| $(0,0) \rightarrow E_{k_u^1} (E_{k_v^0} (k_w^0 \parallel 1))$ |
| $(0,1) \rightarrow E_{k_u^1} (E_{k_v^1} (k_w^1 \parallel 0))$ |
| $(1,0) \rightarrow E_{k_u^0} (E_{k_v^0} (k_w^0 \parallel 1))$ |
| $(1,1) \rightarrow E_{k_u^0} (E_{k_v^1} (k_w^0 \parallel 1))$ |



- Advantage

- Evaluator knows external values and therefore which entry to decrypt. Computing the circuit requires just two decryptions per gate (rather than an average of 5 if 0^n is appended to plaintext)

Yao's protocol

- P_1 sends to P_2
 - Tables encoding each circuit gate.
 - The keys corresponding to P_1 's input values.
- If P_2 gets the keys corresponding to its input values, it can compute the output of the circuit, and nothing else.
 - Why can't P_1 provide P_2 with the keys corresponding to both 0 and 1 for P_2 's input wires?

Yao's protocol

- For every wire i of P_2 's input:
 - The parties run an OT protocol
 - P_2 's input is her input bit (y_i).
 - P_1 's input is k_i^0, k_i^1
 - P_2 learns $k_i^{y_i}$
- The OTs for all input wires can be run in parallel.
- Afterwards P_1 can compute the circuit by itself.

Yao's Protocol

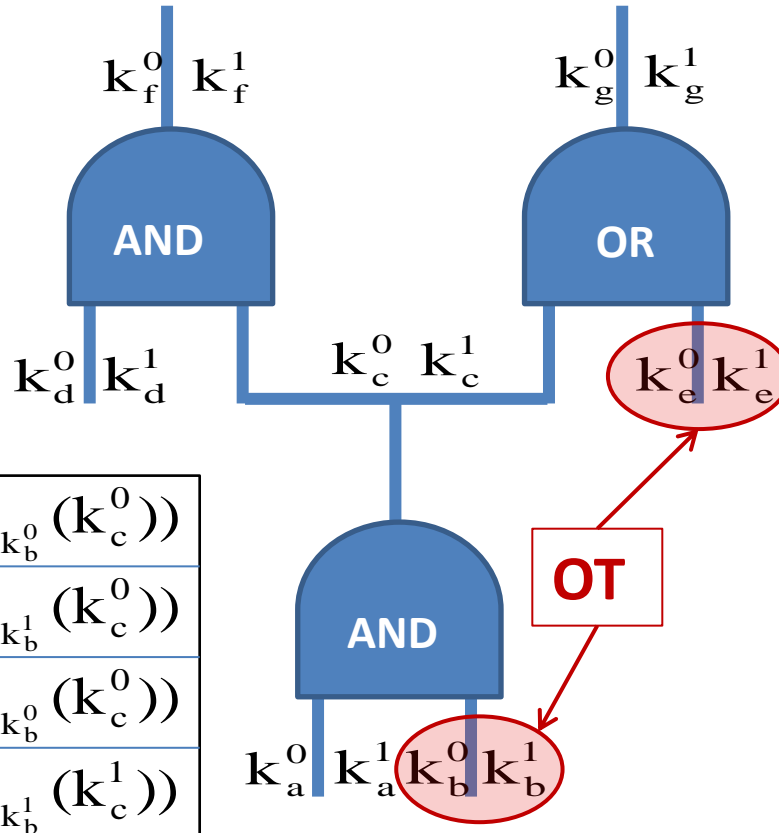
- **Input:** \mathbf{x} and \mathbf{y} of length n
- P_1 generates a garbled circuit $\mathbf{G}(\mathbf{C})$
 - k_L^0, k_L^1 are the keys on wire w_L
 - Let w_1, \dots, w_n be the input wires of P_1 and w_{n+1}, \dots, w_{2n} be the input wires of P_2
- P_1 sends to P_2 $\mathbf{G}(\mathbf{C})$ and the strings $k_1^{x_1}, \dots, k_n^{x_n}$
- P_1 and P_2 run n OTs in parallel
 - P_1 inputs (k_{n+i}^0, k_{n+i}^1)
 - P_2 inputs y_i
- Given all keys, P_2 computes $\mathbf{G}(\mathbf{C})$ and obtains $\mathbf{C}(\mathbf{x}, \mathbf{y})$
 - P_2 sends result to P_1

The Example Circuit

(input wires $P_1 = d, a$; $P_2 = b, e$)

$[(0, k_f^0), (1, k_f^1)]$ $[(0, k_g^0), (1, k_g^1)]$

$E_{k_d^0}(E_{k_c^0}(k_f^0))$
$E_{k_d^0}(E_{k_c^1}(k_f^0))$
$E_{k_d^1}(E_{k_c^0}(k_f^0))$
$E_{k_d^1}(E_{k_c^1}(k_f^1))$



$E_{k_c^0}(E_{k_e^0}(k_g^0))$
$E_{k_c^0}(E_{k_e^1}(k_g^1))$
$E_{k_c^1}(E_{k_e^0}(k_g^1))$
$E_{k_c^1}(E_{k_e^1}(k_g^1))$

$E_{k_a^0}(E_{k_b^0}(k_c^0))$
$E_{k_a^0}(E_{k_b^1}(k_c^0))$
$E_{k_a^1}(E_{k_b^0}(k_c^0))$
$E_{k_a^1}(E_{k_b^1}(k_c^1))$

Double-Encryption Security

- Need to formally prove that given 4 encryptions of a garbled gate and only 2 keys
 - Nothing is learned beyond one output
- Actually, in order to simulate the protocol, we need something stronger
- Notation:
 - Double encryption: $\bar{E}(k_u, k_v, m) = E_{k_u}(E_{k_v}(m))$
 - Oracles: $\bar{E}(\cdot, k_v, \cdot), \bar{E}(k_u, \cdot, \cdot)$

Double-Encryption Security

$\text{Expt}_{\mathcal{A}}^{\text{double}}(n, \sigma)$

1. The adversary \mathcal{A} is invoked upon input 1^n and outputs two keys k_0 and k_1 of length n and two triples of messages (x_0, y_0, z_0) and (x_1, y_1, z_1) where all messages are of the same length.
2. Two keys $k'_0, k'_1 \leftarrow G(1^n)$ are chosen for the encryption scheme.
3. \mathcal{A} is given the challenge ciphertext $\langle \overline{E}(k_0, k'_1, x_\sigma), \overline{E}(k'_0, k_1, y_\sigma), \overline{E}(k'_0, k'_1, z_\sigma) \rangle$ as well as oracle access to $\overline{E}(\cdot, k'_1, \cdot)$ and $\overline{E}(k'_0, \cdot, \cdot)$
4. \mathcal{A} outputs a bit b and this is taken as the output of the experiment.

Enabling \mathcal{A} to access these oracles gives it more power

The encryption is secure if the adversary cannot identify which one of the two triples as encrypted

Proof of Security – P_1 is Corrupted

- P_1 's view consists only of the messages it receives in the oblivious transfers
- In the OT-hybrid model, P_1 receives no messages in the oblivious transfers
- Simulation:
 - Generate an empty transcript

Proof of Security – P_2 is Corrupted

- **More difficult case**

- Need to construct a fake garbled circuit $\mathbf{G}(C')$ that looks indistinguishable to $\mathbf{G}(C)$
- Simulated view contains keys to input wires and $\mathbf{G}(C')$
- $\mathbf{G}(C')$ together with the keys computes $\mathbf{f}(x,y)$
- But the simulator does not know x , so cannot generate a real garbled circuit

Proof of Security – P_2 is Corrupted

- **The simulator**

- Given \mathbf{y} and $\mathbf{z} = \mathbf{f}(\mathbf{x}, \mathbf{y})$, construct a fake garbled circuit $\mathbf{G}'(\mathbf{C})$ that always outputs \mathbf{z}

- Do this by choosing wire keys as usual, but encrypting the **same output key** in all ciphertexts, e.g.

$$\begin{array}{cc} E_{k_u^1}(E_{k_v^0}(k_w^0)) & E_{k_u^1}(E_{k_v^1}(k_w^0)) \\ E_{k_u^0}(E_{k_v^1}(k_w^0)) & E_{k_u^0}(E_{k_v^0}(k_w^0)) \end{array}$$

- This ensures that no matter the input, the same known garbled values on the output wires are received

Proof of Security – P_2 is Corrupted

- **Simulator (continued)**

- Simulation of output translation tables

- Let \mathbf{k}, \mathbf{k}' be the keys on the i^{th} output wire; let \mathbf{k} be the key encrypted in all 4 entries of the gate which outputs this wire
- If $z_i = 0$, write $[(0, \mathbf{k}), (1, \mathbf{k}')]]$
- If $z_i = 1$, write $[(0, \mathbf{k}'), (1, \mathbf{k})]$

- Simulation of input keys phase

- Input wires associated with P_1 's input: send any one of the two keys on the wire
- Input wires associated with P_2 's input: simulate output of OT to be any one of the two keys on the wire

Proof of Security – P_2 is Corrupted

- Need to prove that the simulation is indistinguishable from the real execution
- First step – modify simulator as follows
 - Given circuit inputs x and y (just for the sake of the proof), label all keys on the wires as active or inactive
 - active: key is obtained on this wire upon inputs (x,y)
 - inactive: key is not obtained on wire upon inputs (x,y)
 - Make sure that the single key encrypted in each gate is the active one
- This simulation is identical to the previous one

Proof of Security – P_2 is Corrupted

- Proven by a hybrid argument
 - Consider a garbled circuit $G_L(C)$ for which:
 - The first L gates are generated as in the (alternative) simulation
 - The rest of the gates are generated honestly
- Claim: $G_{L-1}(C)$ is indistinguishable from $G_L(C)$
- Proof:
 - Difference is in L^{th} gate
 - Intuition: use indistinguishability of encryptions to say that cannot distinguish real garbled gate from one where the same active key is encrypted in all entries

Proof of Security – P_2 Corrupted

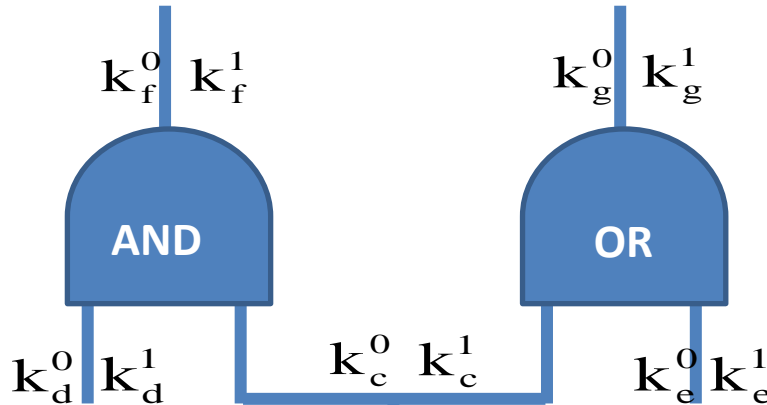
- Observation – L^{th} gate
 - The encryption under both active keys is identical in both cases
 - The difference is encryptions where one or both of the keys are inactive keys
 - Must show that these three encryptions are indistinguishable from the encryptions in real execution
- The problem
 - The inactive keys in this gate may appear in other gates as well
 - We needed the oracles to generate these other encryptions...

The Example Circuit

(input wires $P_1 = d, a$; $P_2 = b, e$)

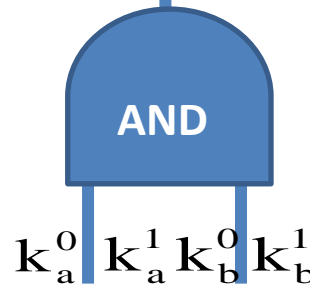
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$E_{k_c^0}(E_{k_e^0}(k_g^0))$
$E_{k_c^0}(E_{k_e^1}(k_g^1))$
$E_{k_c^1}(E_{k_e^0}(k_g^1))$
$E_{k_c^1}(E_{k_e^1}(k_g^1))$

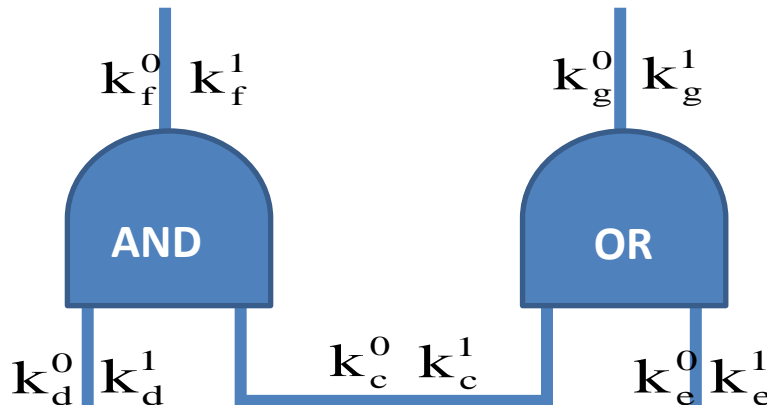
$E_{k_a^0}(E_{k_b^0}(k_c^0))$
$E_{k_a^0}(E_{k_b^1}(k_c^0))$
$E_{k_a^1}(E_{k_b^0}(k_c^0))$
$E_{k_a^1}(E_{k_b^1}(k_c^1))$



Simulator's Circuit (Output 01)

$$[(0, k_f^0), (1, k_f^1)] \quad [(0, k_g^0), (1, k_g^1)]$$

$E_{k_d^0}(E_{k_c^0}(k_f^0))$
$E_{k_d^0}(E_{k_c^1}(k_f^0))$
$E_{k_d^1}(E_{k_c^0}(k_f^0))$
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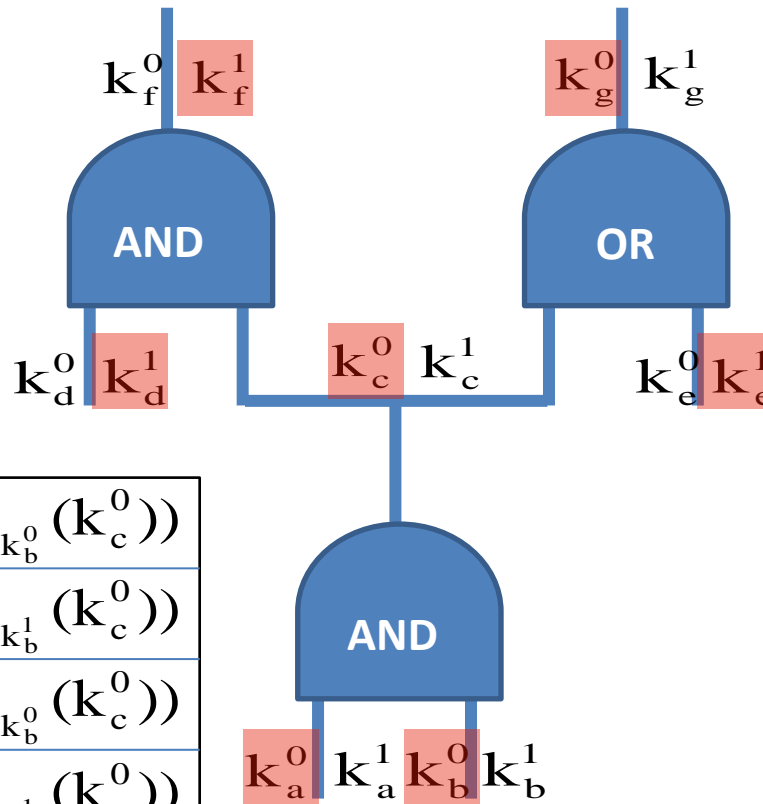
In each gate, all table entries are identical

Inactive Keys

Assuming input is (da=01, be=10), output is (fg=01)

$$[(0, k_f^0), (1, k_f^1)] \quad [(0, k_g^0), (1, k_g^1)]$$

$E_{k_d^0}(E_{k_c^0}(k_f^0))$
$E_{k_d^0}(E_{k_c^1}(k_f^0))$
$E_{k_d^1}(E_{k_c^0}(k_f^0))$
$E_{k_d^1}(E_{k_c^1}(k_f^0))$



$E_{k_c^0}(E_{k_e^0}(k_g^1))$
$E_{k_c^0}(E_{k_e^1}(k_g^1))$
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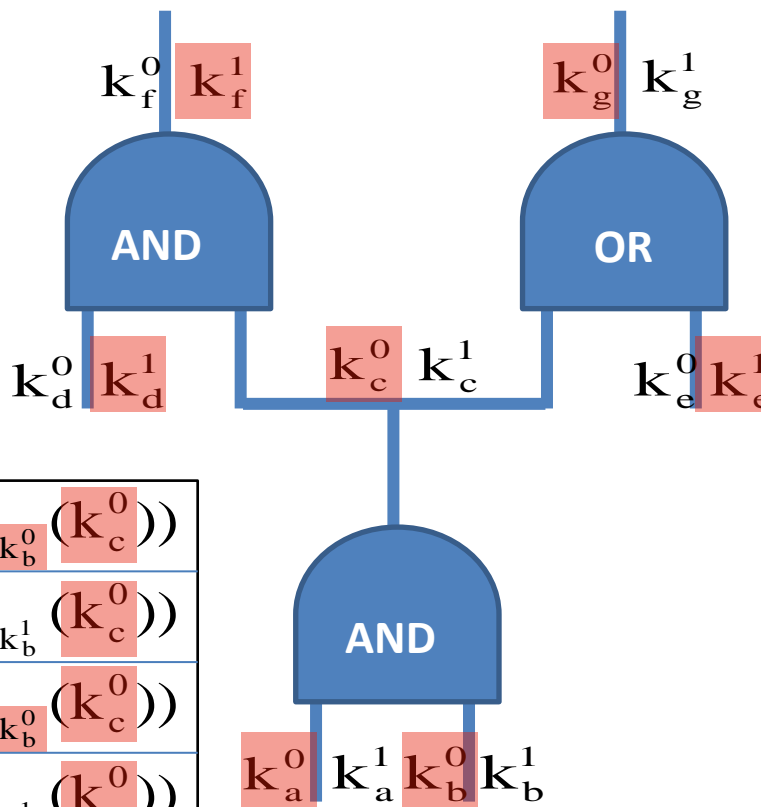
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Inactive Keys

Assuming input is (da=01, be=10), output is (fg=01)

$$[(0, k_f^0), (1, k_f^1)] \quad [(0, k_g^0), (1, k_g^1)]$$

$E_{k_d^0}(E_{k_c^0}(k_f^0))$
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$E_{k_c^0}(E_{k_e^1}(k_g^1))$
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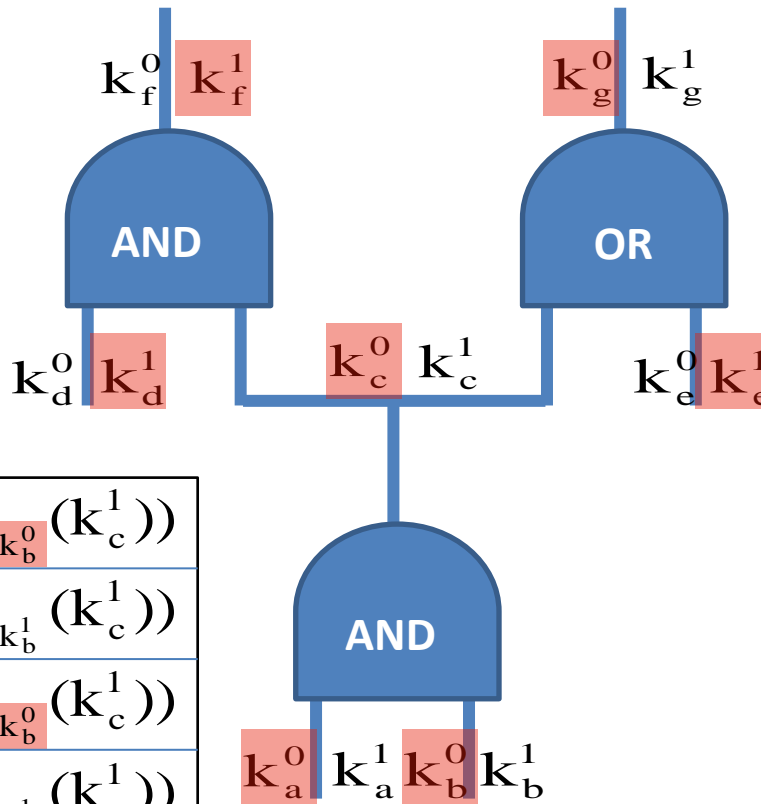
$E_{k_a^0}(E_{k_b^0}(k_c^0))$
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$E_{k_a^1}(E_{k_b^0}(k_c^0))$
$E_{k_a^1}(E_{k_b^1}(k_c^0))$

Modify Simulator

(Encrypt Active Keys Only)

$$[(0, k_f^0), (1, k_f^1)] \quad [(0, k_g^0), (1, k_g^1)]$$

$E_{k_d^0}(E_{k_c^0}(k_f^0))$
$E_{k_d^0}(E_{k_c^1}(k_f^0))$
$E_{k_d^1}(E_{k_c^0}(k_f^0))$
$E_{k_d^1}(E_{k_c^1}(k_f^0))$



$E_{k_c^0}(E_{k_e^0}(k_g^1))$
$E_{k_c^0}(E_{k_e^1}(k_g^1))$
$E_{k_c^1}(E_{k_e^0}(k_g^1))$
$E_{k_c^1}(E_{k_e^1}(k_g^1))$

$E_{k_a^0}(E_{k_b^0}(k_c^1))$
$E_{k_a^0}(E_{k_b^1}(k_c^1))$
$E_{k_a^1}(E_{k_b^0}(k_c^1))$
$E_{k_a^1}(E_{k_b^1}(k_c^1))$

Note
change in
encrypted
key

Hybrid on OR Gate – Simulated OR

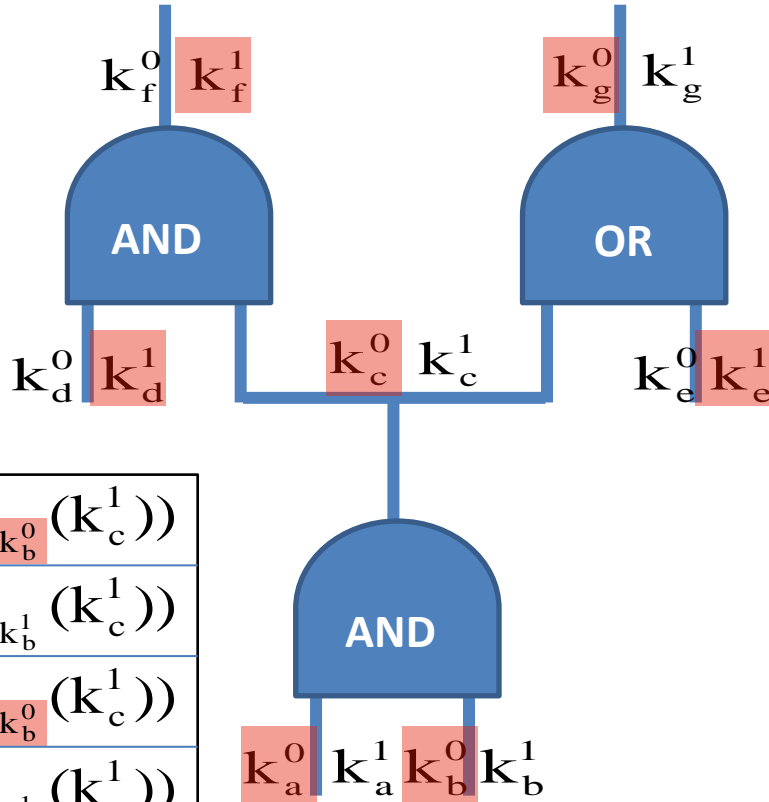
REAL

$$[(0, k_f^0), (1, k_f^1)]$$

$$[(0, k_g^0), (1, k_g^1)]$$

SIM

$E_{k_d^0}(E_{k_c^0}(k_f^0))$
$E_{k_d^0}(E_{k_c^1}(k_f^0))$
$E_{k_d^1}(E_{k_c^0}(k_f^0))$
$E_{k_d^1}(E_{k_c^1}(k_f^1))$



$E_{k_c^0}(E_{k_e^0}(k_g^1))$
$E_{k_c^0}(E_{k_e^1}(k_g^1))$
$E_{k_c^1}(E_{k_e^0}(k_g^1))$
$E_{k_c^1}(E_{k_e^1}(k_g^1))$

SIM

$E_{k_a^0}(E_{k_b^0}(k_c^1))$
$E_{k_a^0}(E_{k_b^1}(k_c^1))$
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$E_{k_a^1}(E_{k_b^1}(k_c^1))$

Hybrid on OR Gate – Real OR

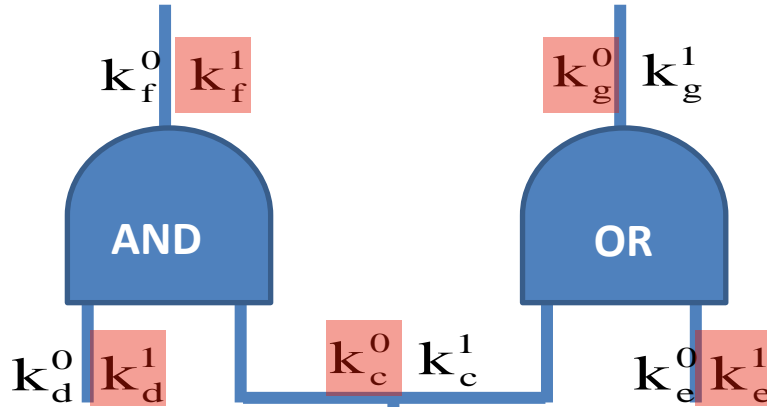
REAL

$$[(0, k_f^0), (1, k_f^1)]$$

$$[(0, k_g^0), (1, k_g^1)]$$

REAL

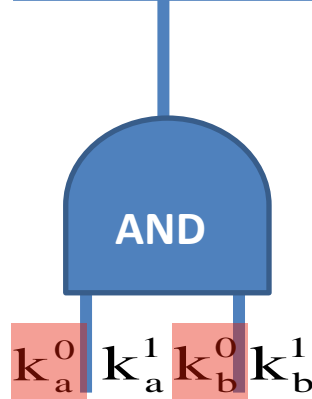
$E_{k_d^0}(E_{k_c^0}(k_f^0))$
$E_{k_d^0}(E_{k_c^1}(k_f^0))$
$E_{k_d^1}(E_{k_c^0}(k_f^0))$
$E_{k_d^1}(E_{k_c^1}(k_f^1))$



$E_{k_c^0}(E_{k_e^0}(k_g^0))$
$E_{k_c^0}(E_{k_e^1}(k_g^1))$
$E_{k_c^1}(E_{k_e^0}(k_g^1))$
$E_{k_c^1}(E_{k_e^1}(k_g^1))$

SIM

$E_{k_a^0}(E_{k_b^0}(k_c^1))$
$E_{k_a^0}(E_{k_b^1}(k_c^1))$
$E_{k_a^1}(E_{k_b^0}(k_c^1))$
$E_{k_a^1}(E_{k_b^1}(k_c^1))$



What's the Difference

- In the **simulated** OR case, the inactive key k_c^0 encrypts the key k_g^1
- In the **real** OR case, the inactive key k_c^0 encrypts the key k_g^0
- Indistinguishability follows from the indistinguishability of encryptions under the **inactive key** k_c^0

Proving Indistinguishability

- Follows from the indistinguishability of encryptions under the **inactive key** k_c^0
- **The good news**
 - Key k_c^0 is not encrypted anywhere (as data) because prior gates are simulated
- **The bad news**
 - The key k_c^0 needs to be used to construct the real AND gate for the hybrid
- **The solution**
 - The special double-encryption CPA game

The problem: inactive key used in another gate

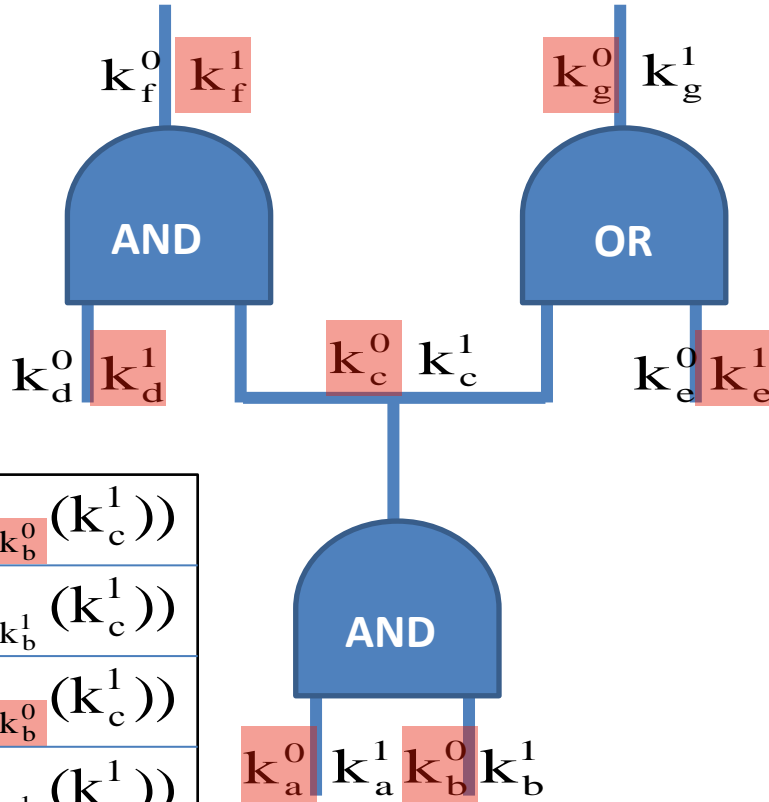
REAL

$$[(0, k_f^0), (1, k_f^1)]$$

$$[(0, k_g^0), (1, k_g^1)]$$

SIM

$E_{k_d^0}(E_{k_c^0}(k_f^0))$
$E_{k_d^0}(E_{k_c^1}(k_f^0))$
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$E_{k_c^0}(E_{k_e^0}(k_g^1))$
$E_{k_c^0}(E_{k_e^1}(k_g^1))$
$E_{k_c^1}(E_{k_e^0}(k_g^1))$
$E_{k_c^1}(E_{k_e^1}(k_g^1))$

SIM

$E_{k_a^0}(E_{k_b^0}(k_c^1))$
$E_{k_a^0}(E_{k_b^1}(k_c^1))$
$E_{k_a^1}(E_{k_b^0}(k_c^1))$
$E_{k_a^1}(E_{k_b^1}(k_c^1))$



Double-Encryption Security

$\text{Expt}_{\mathcal{A}}^{\text{double}}(n, \sigma)$

1. The adversary \mathcal{A} is invoked upon input 1^n and outputs two keys k_0 and k_1 of length n and two triples of messages (x_0, y_0, z_0) and (x_1, y_1, z_1) where all messages are of the same length.
 2. Two keys $k'_0, k'_1 \leftarrow G(1^n)$ are chosen for the encryption scheme.
 3. \mathcal{A} is given the challenge ciphertext $(\overline{E}(k_0, k'_1, x_\sigma), \overline{E}(k'_0, k_1, y_\sigma), \overline{E}(k'_0, k'_1, z_\sigma))$ as well as oracle access to $\overline{E}(\cdot, k'_1, \cdot)$ and $\overline{E}(k'_0, \cdot, \cdot)$.⁵
 4. \mathcal{A} outputs a bit b and this is taken as the output of the experiment.
- k_0, k_1 (i.e., k_c^1, k_e^0) are active keys
 - k'_0, k'_1 (i.e., k_c^0, k_e^1) are inactive keys
 - Can use oracle to generate the REAL AND gate

Proof of Security – P_2 is Corrupted

- Since each gate-replacement is indistinguishable, using a hybrid argument we have that the distributions are indistinguishable (see paper for details)
- QED

Efficiency

- 2-4 rounds (depending on OT and if one party or both parties receive output)
- $|y|$ oblivious transfers
- $8|C|$ symmetric encryptions to generate circuit and $2|C|$ to compute it (using the signal bit)
- For a circuit of 33,000 gates, about 528 Kbytes with 128bit AES encryption

Malicious Adversaries

- **Assume that the OT is secure for malicious adv:**
 - A corrupted P_1 cannot learn **anything** (it receives no messages in the protocol, in the hybrid-OT model)
 - Thus, we have **privacy**
 - We can prove **full security** for the case of a corrupted P_2
- **This can be useful, but...**
 - This does not ensure that the parties compute the required functionality
 - E.g., consider P_1 that builds circuit so that if P_2 's first bit is 0, the circuit doesn't decrypt
 - If P_1 can detect this in the real world, privacy is lost
 - Proving full security against a malicious P_1 is hard

The BMR Protocol

The BMR protocol

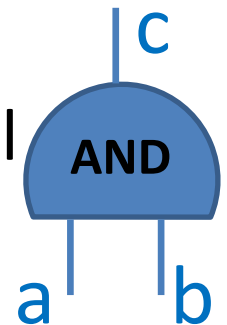
- Beaver-Micali-Rogaway
- A multi-party version of Yao's protocol
- Works in $O(1)$ communication rounds, regardless of the depth of the Boolean circuit. (The GMW, BGW, CCD protocol have $O(d)$ rounds)
 - D. Beaver, S. Micali and P. Rogaway, “The round complexity of secure protocols”, 1990.
 - A. Ben-David, N. Nisan and B. Pinkas, “FairplayMP – A System for Secure Multi-Party Computation”, 2010.

The BMR protocol: the basic idea

- Two random seeds (aka keys, garbled values) are set for every wire of the Boolean circuit:
 - Each seed is a **concatenation** of seeds generated by all players and secretly shared among them.
- The parties **securely compute together** a 4x1 table for every gate (in parallel):
 - Given a 0/1 seed to each of the two input wires, the table reveals the seed of the resulting value of the output wire.

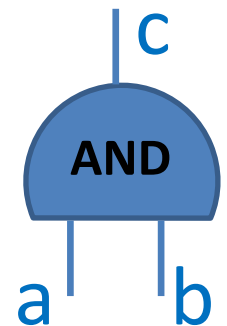
Encoding Gates

- Wire a has seeds $s_{a,1}^0, s_{a,1}^1, \dots, s_{a,n}^0, s_{a,n}^1$ of parties P_1, \dots, P_n .
- Every wire has similar seeds.
- Each wire has a secret bit λ . If $\lambda_a = 0$ then $s_{a,i}^0$ corresponds to an **internal value** of **0** and $s_{a,i}^1$ corresponds to an **internal value** of **1**. Otherwise $s_{a,i}^0$ corresponds to **1** and $s_{a,i}^1$ to **0**.
- The λ values are random and shared between the parties, so no one knows to which internal value the 0 seeds correspond.



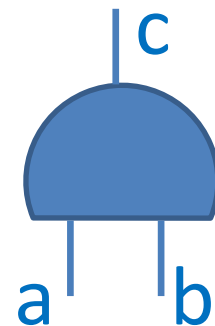
Encoding Gates

- Suppose that $\lambda_a=0$, $\lambda_b=1$ and $\lambda_c=0$.
- The seeds $s_{a,1}^0, \dots, s_{a,n}^0$ and $s_{b,1}^0, \dots, s_{b,n}^0$
 - Correspond to internal values of $a=0$, $b=1$, and consequently to $c=0$.
 - Since $\lambda_c=0$ they will encrypt the corresponding seeds of wire c , $s_{c,1}^0, \dots, s_{c,n}^0$
- Can similarly decide which seed of wire c must be encrypted by each combination of the seeds of wires a, b .



Encoding Gates

- For each gate, the table encrypting the outputs of the gate is a function of
 - $\lambda_a=0, \lambda_b=1, \lambda_c=0$ (these values are shared by the parties)
 - The seeds $s_{a,1}^0, s_{a,1}^1, \dots, s_{a,n}^0, s_{a,n}^1, s_{b,1}^0, s_{b,1}^1, \dots, s_{b,n}^0, s_{b,n}^1,$ and $s_{c,1}^0, s_{c,1}^1, \dots, s_{c,n}^0, s_{c,n}^1$
 - Gate type (AND, OR, etc.)
- The size of this function is independent of the circuit size
- The parties can run a secure computation to compute the table (using, e.g., GMW etc.)



The BMR protocol

- Offline: The parties **securely compute together** a 4x1 table for every gate (**in parallel for all gates**):
 - This is essentially a secure computation of the table
 - All tables are computed in parallel. Therefore overall $O(1)$ rounds.
 - This is the main bottleneck of the BMR protocol (FairplayMP optimizes this computation).
- Online: Given the tables and the seeds of the input values, compute the circuit as in Yao.

Summary

- Can compute any functionality securely in presence of semi-honest adversaries.
- The Yao and BMR protocols are efficient, for circuits that are not too large.
- Obtaining security against malicious adversaries is hard.