Private Set Intersection

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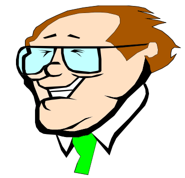
(mostly based on joint work with Thomas Schneider, Gil Segev and Michael Zohner)

Protocols for Specific Problems

- **Generic protocols can securely compute any functionality**
  - Often, the best way to securely compute a function is to represent it as a circuit and apply a generic protocol
  - This is usually the most efficient solution in terms of development time
  - This approach utilizes all improvements that are applied to generic protocols
  - Still, sometimes it is required to achieve better performance than offered by generic protocols

Private Set Intersection (PSI)

Client



Server

	Client	Server
Input:	$X = x_1 \ldots x_n$	$Y = y_1 \ldots y_n$
Output:	$X \cap Y$ only	nothing

Other variants exist (e.g., both parties learn output; client learns size of intersection; compute some other function of the intersection, etc.)

Applications

- **PSI is a very natural problem**
  - **Matching**
    - Testing human genomes [BBC+11]
    - Proximity testing [NTL+11]
  - **Intersection** of suspect lists
    - Botnet detection [NMH+10]
    - Contact list discovery (TextSecure, Secret, Medium)
  - Measuring **conversion rates** for online advertising (Facebook)

This talk

- Survey the major results
- Suggest optimizations based on new observations

- Present new schemes

- Compare the performance of all schemes
  - On the same platform
  - Using the best optimizations that we have

Implementations?

- **Generic circuits seem too large for the job**
  - More about that later

- **PSI is equivalent to oblivious transfer**
  - We'll see PSI protocols based on OT
  - Given PSI we can implement OT:
  - OT: Alice's input is a bit b, Bob's input is two bits x_0,x_1. Alice should learn x_b.
  - Implement OT by computing PSI where
    - Alice uses the input set (b0, b1)
    - Bob uses the input set $(0x_0,1x_1)$

A naïve PSI protocol

- A naïve solution:
  - Have A and B agree on a "cryptographic hash function" $H()$
  - B sends to A: $H(y_1),\ldots, H(y_n)$
  - A compares to $H(x_1),\ldots, H(x_n)$ and finds the intersection

- Does not protect B's privacy if inputs do not have considerable entropy
- This is the algorithm used by all applications we are aware of

Preliminaries

- We only consider semi-honest (passive) adversaries

- Why discuss only semi-honest?
  - There are PSI protocols secure against malicious adversaries [FNP04, JL09, HN10, CKT10, FHNP13]
  - These protocols are much less efficient
  - None of them was implemented

PSI secure against malicious adversaries [FHNP]

$\underline{P_1(X, m_2)}$		$\underline{P_2(Y = \{y_\alpha\}_{\alpha \in \{1 \dots m_2\}}, m_1)}$
$1^k \longrightarrow$ $(pk, sk_1) \longleftarrow$	$\boxed{\pi_{\text{KEY}}}$	$\longleftarrow 1^k$ $\longrightarrow (pk, sk_2)$
	$\xrightarrow{E_{pk}(Q_1(\cdot)) \dots E_{pk}(Q_B(\cdot))}$	
$Q_1(\cdot) \dots Q_B(\cdot) \longrightarrow$	$\boxed{\pi_{\text{POLY}}}$	$\longleftarrow E_{pk}(Q_1(\cdot)) \dots E_{pk}(Q_B(\cdot))$ $\longrightarrow 0/1$
Verify $sk = sk_1 + sk_2$	$\xleftarrow{sk_2}$	
		For all $\alpha \in \{1 \dots m_2\}, j \in \{0,1\}$: $s_0^\alpha, s_1^\alpha \leftarrow_R \mathbb{M}$, $\mathcal{H}(s_j^\alpha) \rightarrow r_j \Vert \tilde{r}_j \Vert \hat{r}_j$ $q_j \stackrel{\text{def}}{=} Q_{h_j(y_\alpha)}(y_\alpha)$
	$e_j^\alpha = E_{pk}(r_j \cdot q_j + s_{1-j}^\alpha; \hat{r}_j)$, $t_\alpha = \tilde{r}_j \oplus y_\alpha$ $\longleftarrow$	
For all $\alpha \in \{1 \dots m_2\}, j \in \{0,1\}$: $s'_j = D_{sk}(\tilde{e}_\alpha)$, $\mathcal{H}(s'_j) \rightarrow r'_j \Vert \tilde{r}'_j \Vert \hat{r}'_j$ Check if $\exists\, x \in X\ j \in \{0,1\}$ s.t. : $t_j^\alpha = \tilde{r}_j \oplus x$, and $\tilde{e}_0^\alpha, e_1^\alpha$, consistent with $r'_1, r'_2, s'_0, s'_1, \hat{r}'_0, \hat{r}'_1$		

Preliminaries – the random oracle model

- In the random oracle model (ROM) a specific function is modeled (in the analysis) as a random function
  - This analysis is very problematic
  - In the theory of crypto, ROM proofs are considered heuristic
- We describe protocols that are based on the ROM
  - There are PSI protocols in the standard model [FNP04], but they are less efficient.
  - We use OT extension
    - Can be based on a non-ROM assumption
    - But the random-OT variant in ROM is even more efficient

Public-key based Protocols

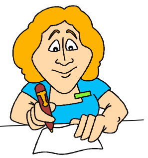
PSI based on Diffie-Hellman

- **The Decisional Diffie-Hellman assumption**
  - Agree on a group G, with a generator g.
  - The assumption: for random a,b,c
    cannot distinguish (g^a, g^b, g^{ab}) from (g^a, g^b, g^c)

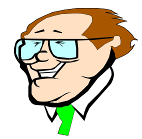
PSI based on Diffie-Hellman

- **The protocol [M86, HFH99, AES03]:**

α



$x_1,\ldots,x_n$



β

$y_1,\ldots,y_n$

in parallel

$(H(x_1))^{\alpha},\ldots,(H(x_n))^{\alpha} \longrightarrow$

$\longleftarrow (H(y_1))^{\beta},\ldots,(H(y_n))^{\beta}$

$\longleftarrow ((H(x_1))^{\alpha})^{\beta},\ldots,((H(x_n))^{\alpha})^{\beta}$

$((H(y_1))^{\beta})^{\alpha},\ldots,((H(y_n))^{\beta})^{\alpha}$

in parallel

Compares the two lists

(H is modeled as a random oracle. Security based on DDH)

Implementation: very simple; can be based on elliptic-curve crypto; minimal communication.

What else could we want?

PSI based on Blind RSA [CT10]

- There is also a PSI protocol based on an RSA variant

- The performance is similar to that of DH based protocols, but
  - In RSA only the owner of the private key does all the hard work $\Rightarrow$ no advantage in the two parties working in parallel
  - Cannot be based on elliptic curve crypto

PSI based on Blind RSA [CT10]

- Bob chooses an RSA key pair $((N,e),d)$
- Alice chooses random $r_1,\dots,r_n$
  computes $x_1\cdot(r_1)^e,\dots, x_n\cdot(r_n)^e$, and sends to Bob.
- Bob computes and sends
  - $H((y_1)^d),\dots, H((y_n)^d)$
  - $(x_1(r_1)^e)^d,\dots, (x_n(r_n)^e)^d$, which equal $(x_1)^d\cdot r_1,\dots,(x_n)^d\cdot r_n$
- Alice divides by r_i, applies $H()$ and compares the lists.

PSI based on Oblivious Polynomial Evaluation [FNP04] (short version)

- (Advantage: proof in the standard model, no ROM)
- Implemented based on additively homomorphic encryption (Paillier, El Gamal).
- Alice generates the polynomial $P(x)=(x-x_1)(x-x_2)\cdots(x-x_n) = a_n x^n + \cdots + a_1 x + a_0$
- Alice sends additively homomorphic encryptions $E(a_0),E(a_1),\ldots,E(a_n)$
- $\forall y_i$ Bob uses these to evaluate and send $E(P(y_i)\cdot r_i+y_i)$
- Implementation: $O(n^2)$ exps. Can be reduced to O(nloglogn) using hashing. Too inefficient.

Generic Protocols

A circuit based protocol

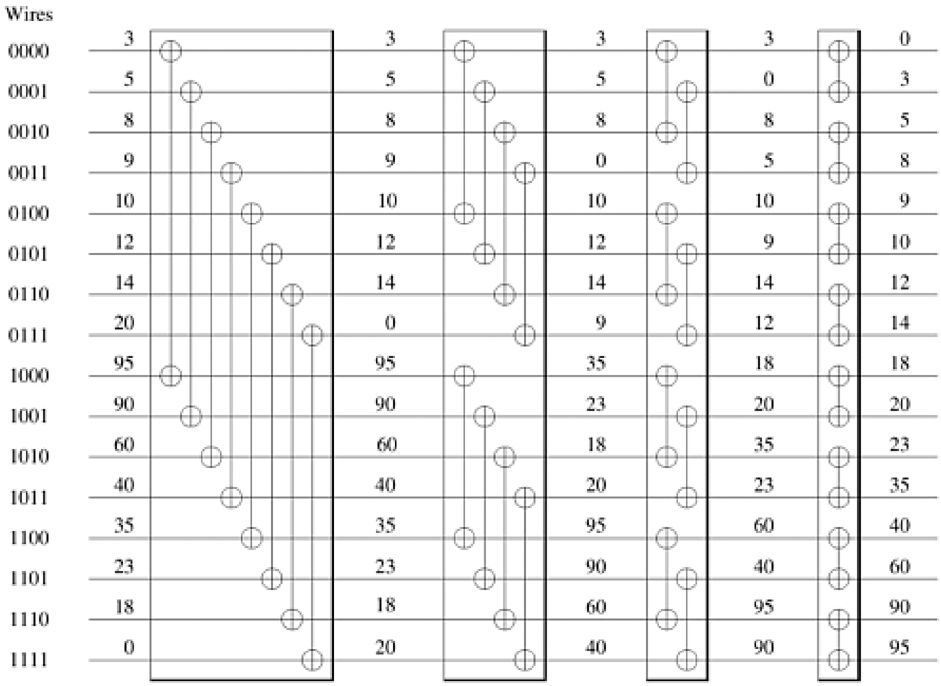
- There are generic protocols for implementing any functionality expressed as a Binary circuit
  - GMW, Yao,…

- A naïve circuit uses n^2 comparisons of words

- Can we do better?

A circuit based protocol [HEK12]

- **A circuit that has three steps**
  - **Sort:** merge two sorted lists using a bitonic merging network [Bat68]. Uses $n\log(2n)$ comparisons.



A circuit based protocol [HEK12]

- **A circuit that has three steps**
  - Sort: merge two sorted lists using a bitonic merging network [Bat68]. Uses $n\log(2n)$ comparisons.
  - Compare: compare adjacent items. Uses $2n$ equality checks.
  - Shuffle: Randomly shuffle results using a Waxman permutation network [W68], using $\sim n\log(n)$ swappings.

  - Overall uses $L \cdot (3n\log n + 4n)$ AND gates. (L is input length)
    - (2/3 of the AND gates are for multiplexers)

Improving Circuit Based PSI

- Initial implementation was done using Yao's protocol
- GMW uses two OTs per gate; Yao uses four symmetric encryptions.
  - Yao was considered much more efficient.
  - OT extension makes GMW faster than Yao.

Recall the evaluation of multiplication gates in GMW

- Input: P_1 has a_1,b_1, P_2 has a_2,b_2.
- P_1 outputs $a_1b_2+a_jb_2+s_{1,2}$. P_j outputs $s_{1,2}$.
- P_j:
  - Chooses a random $s_{1,2}$
  - Computes the four possible outcomes of $a_1b_2+a_2b_1+s_{1,2}$, depending on the four options for P_i's inputs.
  - Sets these values to be its input to a 1-out-of-4 OT implemented using two 1-out-of-2 OT2

Improving Circuit Based PSI

- Note that in the PSI circuit 2/3 of the AND gates are for multiplexers
  - A single bit chooses between two 32 bit inputs
  - For the GMW protocol, instead of independently implementing the OTs for each gate use OTs with inputs that are 32 bit long.
  - It is also possible to implement GMW using *random-OT*, which is more efficient than regular OT.

Performance of Circuit Based PSI

- We will see that circuit based PSI performs unfavorably compared to other protocols

- The main advantage of circuit based PSI is that it can be used to compute any variant of PSI
  - This can be done by a programmer. Other PSI protocols require a *cryptographer* in order to apply any change to the computed function.

PSI based on OT

- OT extension is extremely efficient

- Design simple protocols based on OT
- Use OT extension and hashing based constructions to maximize their performance

First step: Private equality test

- Private equality test
  - Input: Alice has x, Bob has y. Each is s bits long.
  - Output: is x=y?

Private equality test

- Alice input: 001   Bob input: 011

Private equality test

- Alice input: 001   Bob input: 011.
- Random OTs

Alice	
$R_{0,0}$	
$R_{1,0}$	
	$R_{2,1}$

Bob	
$R_{0,0}$	$R_{0,1}$
$R_{1,0}$	$R_{1,1}$
$R_{2,0}$	$R_{2,1}$

Private equality test

- Alice input: 001   Bob input: 011
- Random OTs

Alice

$R_{0,0}$	
$R_{1,0}$	
	$R_{2,1}$

Bob

$R_{0,0}$	$R_{0,1}$
$R_{1,0}$	$R_{1,1}$
$R_{2,0}$	$R_{2,1}$

- Bob sends $R_{0,0} \oplus R_{1,1} \oplus R_{2,1}$
- Alice computes $R_{0,0} \oplus R_{1,0} \oplus R_{2,1}$, and compares.
- Inputs of length s. Random strings of length λ .

Private equality test

- Correctness?
- Security?
- Efficiency?
  – For inputs of length s, run s random OTs of λ bits strings
  – Bob sends a single λ bits string to Alice
  – OTs can be implemented very efficiently using OT extension

Private set inclusion

- Input: Alice has x, Bob has $y_1,\ldots,y_n$
- Output: is x in $\{y_1,\ldots,y_n\}$?

- Run n Private Equality Tests in parallel.
  - Alice's OT choices for all $y_1,\ldots,y_n$ are the same
  - Run only s random OTs of seeds
  - Use a pseudo-random generator to generate from each seed n strings of length λ bits (for the corresponding locations in all columns) ☺
  - Send λn bits from Bob to Alice

Private set intersection

- Input: Alice has $\{x_1,\ldots,x_n\}$, Bob has $y_1,\ldots,y_n$
- Output: Intersection of $\{x_1,\ldots,x_n\}$ and $\{y_1,\ldots,y_n\}$

- Run n Private Set Inclusion protocols

  - Total communication is $\underline{n^2 \lambda}$ bits

  - Communication can be further reduced via hashing

Hashing

- Suppose each party uses a random hash function H(), (known to both) to hash its n items to n bins.
  - Then obviously if Alice and Bob have the same item, both of them map it to the same bin.
  - Each bin is expected to have $O(1)$ items
  - The items mapped to the bin can be compared using private equality tests, with $O(\lambda)$ communication.
  - Overall only $O(n\lambda)$ communication.
- The problem
  - Some bins have more items
  - Must hide how many items were mapped to each bin

Hashing

- Solution
  - Pad each bin with dummy items
  - so that all bins are of the size of the most populated bin

- Mapping n items to n bins
  - The expected size of a bin is $O(1)$
  - The maximum size of a bin is whp $O(\log n)$
  - Communication increases by $O(\log n)$ to be $O(n\lambda\log n)$ ☹

Hashing

- Mapping n items to about $n / \ln n$ bins
  - The expected size of a bin is $\approx O(\ln n)$
  - The maximum size of a bin is (whp) the same
  - This is ideal, since we cannot hope to pay less than the expected cost

Other hashing schemes

- Power of two hashing (balanced allocations)
- Cuckoo hashing

	Total #OTs	OT comm.	Overall Comm. (MB) for $n=2^{18}$
No hashing	*ns*	$n^2\lambda$	327,808
Simple hashing	3.7ns	$n\lambda$	475
Balanced hashing	2.9ns lnlnn	$2n\lambda$	939
Cuckoo hashing	$(2(1+\varepsilon)n+\ln n)s$	$(2+\ln n)n\lambda$	276

Input length

- The protocol performs an OT for each bit in the representation of the input items
- Reducing input length $\Rightarrow$ reducing overhead!

Hashing: can inputs be shorter?

- When mapping n items to n/lnn bins each bin has O(ln n) items.
  - Birthday paradox: Can hash down input values to O(lnln n) bits, and expect no collisions in a bin!
  - $N=2^{20} \Rightarrow$ lnln n = 2.6. Wow!!!
  - Unfortunately, to obtain an error probability of 2^{-s} in the birthday paradox, one needs to represent each item using s+lnlnn bits.
  - For reasonable error probabilities we gain nothing ☹

Permutation based Hashing [ANS,PSSZ15]

- Hash the values in the bins to a shorter representation while ensuring that different values map to different hashes.
  - Assume we have 2^b bins. Input length is $|x| > b$.
  - $x = x_L x_R$, where $|x_L| = b$.
  - f is a random function whose range is $[1, 2^b]$.
  - x is mapped to bin $x_L \oplus f(x_R)$.
  - Store in that bin the value x_R.

Permutation based Hashing [ANS,PSSZ15]

- Hash the values in the bins to a shorter representation while ensuring that different values map to different hashes.
  - Assume we have 2^{20} bins. Input length is $|x| = 32$.
  - $x = x_L x_R$, where $|x_L| = 20$.
  - f is a random function whose range is $[1, 2^{20}]$.
  - x is mapped to bin $x_L \oplus f(x_R)$.
  - Store in that bin 12 bits.

Permutation based Hashing [ANS,PSSZ15]

- Hashing is Feistel like
  - x is mapped to bin $x_L \oplus f(x_R)$.
  - Store in the bin the value x_R.
- If x,x' are mapped to the same bin and store there the same value, then x=x', since
  - Same value: $x_R = x'_R$
  - Same bin: $x_L \oplus f(x_R) = x'_L \oplus f(x'_R)$

Permutation based Hashing

- Great savings!
  - Assume $|x|=32$ and $2^b=2^{20}$ bins.
  - Permutation-based hashing stores in a bin the value x_R of length 12 bits (instead of 32 bits).
  - The overhead of the protocol is reduced to about 12/32 = 37.5% of original cost!
  - Will see performance results in a minute

Generic Computation + Permutation Based Hashing [PSSZ15]

- PSI based on generic secure computation + permutation based hashing
  - Alice maps her inputs to bins (using Cuckoo hashing)
  - Bob maps his inputs to bins
  - They both use permutation-based hashing to reduce the length of their input representations
  - For each bin, they evaluate a circuit that simply compares the elements mapped to it by both parties

Generic Computation + Permutation Based Hashing [PSSZ15]

- Advantages
  - SCS circuits compare all input elements to each other. The new circuits work independently on each bin and use shorter representations.
  - For representation length σ, the entire new circuit has $n\sigma\log n$ non-xor gates, and a depth of only $\log \sigma$. (SCS has $O(n\sigma'\log n)$ gates, and depth $O(\log n \log \sigma')$.)
  - The depth affects number of communication rounds…
  - The circuit is very regular: this reduces memory footprint and enables easy parallelization.

Experiments

- No previous “fair” comparison of all protocols

- We used two desktops in a LAN and cloud settings
  - Inputs are 32 bit long
  - Statistical security parameter λ=40
  - Symmetric security parameter of 128 bits

Experiments: run time msec (for 2^{16} items)

Protocol	local	cloud
Naïve insecure hashing	48	560
DH ECC	51,400	162,000
Sorting circuit	47,700	225,500
Perm-based hash circuit	10,500	42,500
Perm-based hash + OT	442	3000

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For $n=2^{20}$ items run time of insecure hashing is 710msec, and of the Perm-based hash + OT based protocol 4500msec.
Ratio of about 6.3
For $n=2^{24}$ items the ratio is about 3.4

Experiments: run time msec (for 2^{16} items)

Protocol	local	cloud
Naïve insecure hashing	48	560
DH ECC	51,400	162,000
Sorting circuit	47,700	225,500
Perm-based hash circuit	10,500	42,500
Perm-based hash + OT	442	3000

The permutation-based hashing circuit is about 4-5 times faster than sorting based circuits.
Still, circuits are slower than other solutions.

Experiments: run time msec (for 2^{16} items)

Protocol	local	cloud
Naïve insecure hashing	48	560
DH ECC	51,400	162,000
Sorting circuit	47,700	225,500
Perm-based hash circuit	10,500	42,500
Perm-based hash + OT	442	3000

The Diffie-Hellman protocol is slow, but is as far the easiest to implement.

Communication in MB (2^{16} items)

Protocol	
Naïve insecure hashing	0.55
DH ECC	4.5
Sorting circuit	3,300
Permutation based circuit	1,050
Perm-based hash + OT	6.5

The Diffie-Hellman protocol has the best communication. The Perm. based hash + OT protocol is pretty close.

Conclusions

- Set intersection can be efficiently applied to very large input sets
- Different settings require different protocols
  - Run time
  - Communication
  - Generality
  - Development time
- Nice combination of crypto/hashing/systems research.